

Towards a spin dual of the fractional quantum Hall effect

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Abstract

Electromagnetic duality between the Aharonov-Bohm and the Aharonov-Casher quantum mechanical phases predicts the existence of a new collective state of matter which can be regarded as a spin dual to the fractional quantum Hall effect. The state, induced by electric fields, is driven by effective spin-spin interactions. We derive experimental and materials conditions of spin-spin interactions and electric fields under which the new state may be observed.

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Electromagnetic fields can influence quantum mechanical systems by generating quantum phases. The Aharonov-Bohm (AB) effect [1] describes the quantum phase generated by a magnetic field enclosed by the trajectory of an electrical charge. A dual to the AB effect exists, whereby an electric field generates a phase along the trajectory of a magnetic moment. This quantum phase is referred to as the Aharonov-Casher (AC) phase [2]. The concept of electromagnetic duality has been fruitfully utilized in modern quantum field theory to study the otherwise intractable strongly-coupled limit of various theories, starting from better understood weakly-coupled ones [3]. The approach has illuminated how apparently disconnected problems are related as respectively strongly and weakly-coupled limits of the same core phenomena. In this letter we explore an application of duality to correlated electron systems and address whether the approach may lead to new collective states of matter and observable effects, specifically akin to the fractional quantum Hall effect (FQHE) [4]. We find that the AC phase carries deep implications for phenomena in correlated electron systems as likewise the AB phase has proven to significantly impact charge transport in the solid state. Indeed, signatures of the AB phase abound in the solid state, leading to phenomena such as oscillatory effects in mesoscopic rings [5], weak-localization [6], universal magnetoconductance fluctuations [7], the creation of composite particles in the FQHE [8], and flux quantization in superconductors.

The role of the AB phase in the FQHE is well known, and hence the study of a dual to the FQHE can start with an exploration of the dual phase, the AC phase. The expression below for the AC phase emphasizes the duality with the AB phase (in SI units, AC phase at left, AB phase at right):

$$\Delta\phi_{AC} = \frac{1}{\hbar c^2} \int_C \vec{\mu} \cdot (\vec{E} \times d\vec{l}), \quad \Delta\phi_{AB} = \frac{1}{\hbar} \int_C q(\vec{A} \cdot d\vec{l}). \quad (1)$$

Here $\Delta\phi_{AC}$ denotes the AC phase, $\Delta\phi_{AB}$ the AB phase, $\vec{\mu}$ the particle's magnetic moment, q the particle charge, \vec{E} the electric field, \vec{A} the magnetic vector potential, $d\vec{l}$ a line element of the trajectory, and c the velocity of light. The ring geometries in Fig.1 help in visualizing the AC and AB effects. For simplicity in the figure and without impinging on generality, $\vec{\mu}$ is assumed perpendicular to \vec{E} . The expression for AC in Eq.1 results from a permutation of the expressions obtained in Ref.2, and can formally be obtained by introducing an effective $q\vec{A}_{eff} = (1/c^2)\vec{\mu} \times \vec{E}$. The duality between the AB and AC effects arises from the topological equivalence of, on one hand, a closed path of a charge q around a local magnetic flux tube and, on the other hand, a closed path of a local magnetic flux around a charge q (generating \vec{E}) [9]. In neither the AB or AC effect do the magnetic field \vec{B} or \vec{E} respectively effect a force [9, 10]. The AC phase implied by Eq.1 is properly a dynamical phase [11], but a Berry's phase may additionally arise if $\vec{\mu}$ evolves over its trajectory [12].

Experimentally, the AC effect was observed using neutron beam interferometry [13]. The duality in the solid state of AB and AC phases was emphasized by Mathur [14] in a theoretical study of antilocalization. Other theoretical studies emphasize the role of the AC phase in interference effects under spin-orbit interaction (SOI) in mesoscopic rings [11, 12]. Experimental efforts so far perform AB-type experiments on mesoscopic rings in a variable perpendicularly applied \vec{B} , approaching the AC phase as a modification to the AB phase under strong SOI, rather than as a dual effect from which new states of matter may arise.

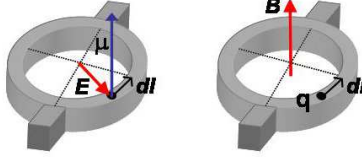


Figure 1: Schematic ring geometries to visualize the duality between the Aharonov-Casher (left) and Aharonov-Bohm (right) effects. Particle paths follow line elements \vec{dl} . For simplicity, the magnetic moment $\vec{\mu}$ is represented perpendicular to the radial electric field \vec{E} .

Ring arrays in InGaAs heterostructures were studied [15], as well as inconclusive single rings on HgTe [16]. Current experiments aiming to demonstrate the AC phase in the geometry of Fig.1 will help demonstrate the duality implicit in Eq.1.

The many-body Hamiltonian describing the FQHE collective state of matter reads:

$$\frac{1}{2m^*} \sum_j [-i\hbar \vec{\nabla}_j - q\vec{A}(\vec{r}_j)]^2 + \sum_{j<k} V_C(\vec{r}_j - \vec{r}_k) \quad (2)$$

where $\vec{\nabla}_j$ represents the gradient with respect to coordinate \vec{r}_j of the j^{th} electron and m^* the effective mass. The pair-wise Coulomb interaction energy V_C causes the FQHE. A potential term expressing a neutralizing background charge has been omitted for simplicity [17]. Similarly a dual may be constructed by substituting $q\vec{A}_{eff} = (1/c^2)\vec{\mu} \times \vec{E}$ and considering the many-body Hamiltonian for interacting $\vec{\mu}_j$ or spins in an applied \vec{E} :

$$\frac{1}{2m^*} \sum_j [-i\hbar \vec{\nabla}_j - (1/c^2)\vec{\mu}_j \times \vec{E}]^2 + \sum_{j<k} V_C(\vec{r}_j - \vec{r}_k) + \sum_{j<k} V_S(\vec{\mu}_j, \vec{\mu}_k) \quad (3)$$

with $V_S(\vec{\mu}_j, \vec{\mu}_k)$ denoting a spin-spin interaction energy if $\vec{\mu}_j = g^* \mu_B S_j$ (here S_j is the particle spin, g^* is the electron g factor in the material, and $\mu_B = e\hbar/2m_e$ is the Bohr magneton). We explore whether dominantly strong spin-spin interactions V_S may in the presence of an applied \vec{E} result in a dual of FQHE, referred to as the spin dual quantum Hall effect (SDQHE). Spin-spin interactions have received substantial recent attention for their role in the pairing mechanisms suggested in superconductivity [18], and can in select systems play a dominant role.

As proposed by Laughlin [19] for two-dimensional systems (2DSs), the ground state wave function for Eq.2 at a FQHE state characterized by odd fractional Landau level filling factor $\nu = 1/(2n+1)$, is:

$$\Psi_{2n+1}(z_1, \dots, z_n) = \prod_{j<k} (z_j - z_k)^{2n+1} \exp\left(-\frac{1}{4l_B^2} \sum_{i=1}^N |z_i|^2\right) \quad (4)$$

where n is an integer, $z_j = x_j + iy_j$ are the complex coordinates of the j^{th} electron, and $l_B = \hbar/eB$ is the magnetic length. The exponential term of the Landau level wave functions is multiplied by the Jastrow factor $(z_j - z_k)^{2n+1}$ by which three functions are fulfilled [20]: 1) the many-body wave function acquires the correct antisymmetry under particle exchange,

and 2) \vec{B} is accounted for with the correct ν , and 3) the correlations due to Coulomb interaction are satisfied. The Jastrow factor attaches to each electron a flux tube of $2n + 1$ magnetic Aharonov-Bohm flux quanta (one flux quantum is $\phi_0 = h/e$), ensuring an overall correct ν for the homogeneously applied \vec{B} . The addition of magnetic flux quanta also decreases the Coulomb interaction energy since the Jastrow factor reduces the wave function amplitude if electrons approach each other. Functions (2) and (3) of the Jastrow factor in Laughlin's wave function embody the core of the FQHE, and are also amenable to the SDQHE. We envision a dual state where an interacting spin system under an applied \vec{E} is described by a Laughlin-type wave function. To build the SDQHE, the magnetic flux quantum in the FQHE is substituted by an electrical line integral quantum, Y_0 defined below (see also Eq.1 and Fig.1) and we assume a spin-spin interaction V_S dominant over the Coulombic V_C . Experimental systems for the realization of this crucial condition are discussed in later sections. For the emergence of the FQHE, the exact form of V_C is not important, as it suffices that the Jastrow factor captures the major part of the interactions, leaving residuals unable to change the ground state. Likewise, we expect latitude in the exact form of V_S causing the SDQHE. The dual described by the corresponding Laughlin wave function is a new correlated Fermionic state, as is understood from the odd exponent in the terms $(z_j - z_k)^{2n+1}$. A description of the FQHE has evolved whereby the FQHE is understood as the integer quantum Hall effect (IQHE) of composite Fermions [20]. Particles similar to the composite Fermions are expected to have significance in the SDQHE, and will likewise possess Fermionic statistics. A spin dual to the IQHE is briefly discussed below.

To establish similarities to quantum Hall geometries we consider a 2DS in the x-y plane. In quantum Hall geometries, $\vec{B} = (0, 0, B)$ may be generated from the symmetric gauge potential $\vec{A} = (-By/2, Bx/2, 0)$. In a gedanken experiment for the SDQHE, we assume the spins $\vec{\mu}$ are aligned parallel to z , resulting from ferromagnetic interactions. From $q\vec{A}_{eff} = (1/c^2)\vec{\mu} \times \vec{E}$ a symmetric gauge linear in coordinates then signifies a radially increasing in-plane \vec{E} , as encountered in a uniformly charged cylinder. In this gedanken experiment, wave functions [21], energy levels and degeneracies of a magnetic moment in \vec{E} faithfully mimic a charge in \vec{B} . For other profiles of \vec{E} , energy levels may not be equally spaced and level degeneracies may differ, without impact, however, on the development of the SDQHE. Without impact on generality while allowing a more transparent treatment, we will below consider the special case of spins oriented along z . Beyond this special case, the consequences of the full $SU(2)$ spin symmetry as well as antiferromagnetic alignment promise a rich spectrum of behavior in the SDQHE. For $\vec{\mu}$ parallel to z , the above similarity ensures that akin to the IQHE, broken translational invariance at sample edges will result in edge states. Arguments of gauge invariance and charge conservation [22] then lead to a spin dual of the IQHE. Since the carrier spin is associated with a charge, electrical transport characteristics akin to the IQHE and FQHE are expected for the corresponding spin duals. The energy level structure is carried by spin, whereas charge conservation preserves the quantization argument. In this broad context, routes distinct from dualization, involving SOI and the role of $SU(2)$ in the quantum Hall effect have been discussed previously [23].

From the geometries in Fig.1 and the argument involving the radial in-plane \vec{E} above, we may derive general experimental geometries in which the SDQHE may be observed. \vec{B}

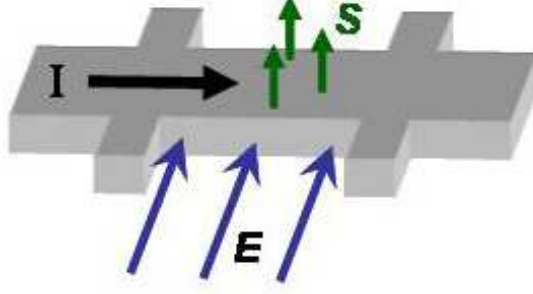


Figure 2: Hall bar geometry with in-plane electric field \vec{E} , perpendicular to current direction (\mathbf{I}). We consider only the special case of spins (S) oriented perpendicularly to \vec{E} .

perpendicular to the plane of the 2DS introduces the flux quanta h/e in the FQHE. The SDQHE will require \vec{E} in the plane of the 2DS, which introduces an electrical line integral quantum, Y_0 . In analogy with the flux periodicity ϕ_0 observed in mesoscopic rings due to the AB phase, an AC periodicity can be expressed by stating:

$$\frac{1}{\hbar c^2} \int_C \vec{\mu} \cdot (\vec{E} \times d\vec{l}) = 2\pi p \quad (5)$$

with p an integer. With the projection of $\vec{\mu}$ perpendicular to the line integral equated to μ_B , the periodicity in \vec{E} is deduced from:

$$Y_0 = \int_C \vec{E} \times d\vec{l} = \frac{4\pi m_e c^2}{e} = 6.42 \times 10^6 V \quad (6)$$

where Y_0 carries in the AC phase the same role as ϕ_0 carries in the AB phase. Other authors have derived quantization conditions for the AC phase, and have expressed the phase as an addition to the AB phase [14, 24]. In vacuum, the AC phase for an electron spin remains small for technically achievable magnitudes of \vec{E} . Even for a macroscopic sample where the line integral courses over a circumference of order 1 mm the vacuum value for Y_0 predicts that very substantial $E \sim 10^{10} V/m$ are necessary, higher than the breakdown field of many electronic materials ($\sim 10^7 V/m$). In materials with high SOI, however, the required E is much reduced [14, 24], providing an avenue for experimental observation of the SDQHE. By considering a small section of material wherein the radial field lines appear approximately parallel (a conformal mapping argument), we conclude that a Hall bar geometry with an in-plane \vec{E} perpendicular to the current direction can support the SDQHE, as illustrated in Fig.2. Under dominant V_S , spin and charge both will interact with the applied \vec{E} , and experiments should be designed to allow the data to distinguish the interactions. Particularly in semiconductors, experiments should also account for gating and the electrostatic AB effect (hitherto not observed) [25].

The FQHE appears in 2DSs where the ratio of the average Coulomb interaction energy $\langle U_C \rangle$ to kinetic energy is large. The parameter $r_C = \langle U_C \rangle / E_F$, where E_F is the Fermi energy, expresses this ratio. In GaAs/AlGaAs 2DSs, $r_C \sim 2...30$, high values allowing the observation of the FQHE. The parameter r_C is for the SDQHE recast as $r_S = \langle U_S \rangle / E_F$

with $\langle U_S \rangle$ the average spin-spin interaction energy. For comparison we first consider magnetic dipole-dipole interactions with parallel spin alignment (achieved by a weak constant \vec{B} parallel to z with B much below the FQHE regime). With values for m^* and g^* typical of semiconductors, we find $r_S \sim 10^{-6}$, a small ratio expected from the higher-order dipole interactions. It is hence advisable to identify systems with stronger spin-spin interactions. Magnetic exchange interactions between itinerant electron or hole spins form promising candidates. The carrier systems of interest should also allow for the application of \vec{E} . Dilute magnetic semiconductors, particularly Mn-doped III-V semiconductors, may unite these properties. Exchange between the Mn spins and the hole spins lead to effective large hole spin-spin interactions of the form $V(\vec{\mu}_j, \vec{\mu}_k) = J_{jk}(\vec{r}) \vec{\mu}_j \cdot \vec{\mu}_k / (g^* \mu_B)^2$. The exchange mechanism, still under discussion, determines the spatial dependence of $J_{jk}(\vec{r})$ [26]. An estimate can be obtained from $r_S \sim k_B T_C / E_F$ with T_C the Curie temperature and k_B the Boltzmann constant. For representative hole densities in Mn-doped III-V semiconductors ($3 \times 10^{26} m^{-3}$) and $T_C \sim 150K$, we derive $r_S \sim 0.1$. While still lower than r_C , this value in 3-dimensional materials heartens experimental efforts. Yet, an experimental complication in dilute magnetic semiconductors may reside in the high concentration ($\sim 1.5\%$) of Mn, resulting in substantial disorder likely deleterious to the SDQHE as it is for the FQHE. In fact, band transport in GaMnAs is questioned [27]. In other systems, at the border of ferromagnetic or antiferromagnetic transitions spin susceptibilities increase and strengthen induced spin-spin interactions. Recent insights in unconventional superconductivity [18] suggest that these increased spin-spin interaction can overcome Coulomb repulsion.

In conclusion duality between the AB and AC phases, when applied to the FQHE, predicts a novel correlated state of matter. The state, a spin dual to the FQHE with signatures akin to the FQHE, may appear under electric fields in new materials with strong spin-spin interactions.

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